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Application of a feature selection technique to samples of high resolution synthetic aperture radar imagery

Richard A. Hevenor

July 1983

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U.S. ARMY CORPS OF ENGINEERS **ENGINEER TOPOGRAPHIC LABORATORIES FORT BELVOIR, VIRGINIA 22060**







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A feature selection technique was applied		
technique was applied to four classes of		
The four classes considered were forests,		
was computed from samples of each class	s. A linear transform	nation was utilized to develop a new
feature vector of reduced dimensionality	y. This transformation	
most effective for performing class separab	oility	

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PREFACE

The authority for performing the work described in this research note is contained in Project 4A161102B52C, "Research in Geodetic, Cartographic, and Geographic Sciences."

The work described in this research note represents an application of a standard feature selection technique to samples of high resolution synthetic aperture radar imagery. The task was performed under the supervision of Dr. Frederick W. Rohde, Team Leader, Center for Physical Sciences and Mr. Melvin Crowell, Jr., Director, Research Institute.

COL Edward K. Wintz, CE, was the Commander and Director and Mr. Robert P. Macchia was the Technical Director of the Engineer Topographic Laboratories during the study period.

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APPLICATION OF A FEATURE SELECTION TECHNIQUE TO SAMPLES OF HIGH RESOLUTION SYNTHETIC APERTURE RADAR IMAGERY

INTRODUCTION

The purpose of this research note is to show the application of a feature selection technique to samples of synthetic aperture radar imagery and to present some preliminary results. In the past, feature selection techniques have been applied to data computed from digitized aerial photography. However, it appears that no one has as yet applied feature selection techniques to high resolution radar imagery. In order to perform classification of terrain features using radar imagery, feature selection is an important initial step. Feature selection consists of choosing those features that are most effective for showing class separability and for performing a reduction in the dimensionality of the feature vector. The following sections will present a discussion of the feature selection technique, along with its application to selected samples of radar imagery.

The application of pattern recognition techniques is accomplished usually in two steps, namely, feature selection and classifier design. The feature selection process that precedes the classification process consists of techniques applicable to one class or to multiple classes. The feature selection technique to be discussed here is applicable to multiple classes. It provides the capability of reducing the number of components of the original feature vector in such a way that the resulting components are optimized to show class separability. The feature selection technique comes from the field of discriminant analysis of statistics and is independent of the probability density functions of the feature vector data. The feature selection operation can be expressed as a linear transformation of the following form:

$$Y = AX \tag{1}$$

where X is the original feature vector with dimensionality $n \times 1$; A is the transformation matrix of dimensionality $m \times n$, where m is less than n; and Y is the transformed feature vector with dimensionality $m \times 1$. The feature selection problem is now reduced to determining the matrix A. In order to calculate A. use is made of the within-class and between-class scatter matrices. A within-class scatter matrix shows the scatter of samples around their class expected vector and can be expressed by

$$S_{w} = \sum_{i=1}^{N} P(\omega_{i})C_{i}$$
 (2)

where S_w is the within-class scatter matrix, $P(\omega_i)$ is the a priori probability of the i^{th} class, C_i is the covariance matrix of the i^{th} class, and N is the total number of classes. A between-class scatter matrix can be defined in many ways; however, the following definition was the one utilized here:

$$S_b = C_1 + C_2 + (M_1 - M_2) (M_1 - M_2)^T$$
 (3)

where S_b is the between-class scatter matrix, C_1 is the covariance matrix for class 1, C_2 is the covariance matrix for class 2, M_1 is the mean vector for class 1, M_2 is the mean vector for class 2, and T means transpose. This definition of the between-class scatter matrix is valid only for the case where N (the number of classes) is equal to two. In order to have criteria for class separability, a number must be derived from the scatter matrices. This number should increase when the distances between points belonging to different classes are increasing or when the distances between points belonging to the same class are decreasing. One criterion is the use of J_1 , which can be defined as follows:

$$J_1 = \text{trace } (S_{2m}^{-1} S_{1m})$$
 (4)

where

$$S_{2m} = AS_w A^T$$
 and $S_{1m} = AS_b A^T$

The feature selection problem now requires that we select the particular transformation matrix A, which maximizes the value of J_1 . Fukunaga¹ shows that A is made up of the normalized eigenvectors of the matrix S_w^{-1} S_b .

$$\mathbf{A}^{\mathrm{T}} = [\phi_1 \ \phi_2 \dots \phi_m] \tag{5}$$

where ϕ_1 is the eigenvector associated with the largest eigenvalue, ϕ_2 is the eigenvector associated with the second largest eigenvalue, etc. Once the matrix A is computed from (5), the new feature vector Y can be computed for each point in each class.

The feature selection technique was applied to samples **INVESTIGATION** of high resolution synthetic aperture radar imagery taken over the Huntsville, Alabama, area with the APD-10 radar system. Sections of the radar imagery were digitized and stored on a digital disk unit. A Lexidata system 3400 display processor was used to display the images on a cathode ray tube and to take 100 samples for each of four terrain classes from the imagery. Each sample consisted of a 32 by 32 pixel element window located within a section of one particular terrain class. The four classes considered were cities (combination of commercial and residential structures, DLMS category \$504 FIC 301 and \$505 FIC 401), fields (agriculture used primarily for crop and pasture land, DLMS category #510 FIC 950), water (rivers with smooth fresh water. DLMS category #510 FIC 940) and fresh water subject to ice (lakes and reservoirs. DLMS category #510 FIC 943), and forests (mixed trees, deciduous and evergreens, DLMS category #510 FIC 954). A feature vector consisting of 13 components was computed for each sample. These 13 components were made up of the first- and second-order gray level histogram statistics computed from each sample window. The explicit equations used for the 13 components of the original feature vector are shown in appendix A. A computer program was written for the Hewlett-Packard 1000 computer to calculate J₁ as a function of displacement in x and y. This computer program was also used to calculate the transformation matrix A. A listing of this program is provided in appendix B. A second computer program was written to calculate the new feature vector Y for the four hundred samples taken from the radar imagery. A listing for this second computer program is given in appendix C.

¹Keinosuke Fukunaga, Introduction to Statistical Pattern Recognition, Academic Press, 1972.

In this section some results of numerical calculations are presented. Table 1 shows the results of calculating the value of J_1 for various values of displacement DX and DY for the two classes of forests and cities. The a priori probabilities for the two classes were assumed equal to $0.5.^2$ In table 1 the largest value of J_1 is 29.8525, which is associated with a DX of -3 and a DY of 4. The significance of a maximum value of J_1 occurring at these particular values of displacement is not understood at this time. The values of the displacement DX and DY associated with the largest value of J_1 were utilized to compute a new feature vector Y with two dimensions.

Figure 1 shows a plot of two second-order statistical components for the forest and city samples. The triangles are the results from city calculations; and t' results from forest calculations. This figure clearly shows the necestor feature selection because the original data for forests and cities is not separ · Figure 2 presents the results after the transformation Y = AX is applied are original feature vector X. In this figure the x's again represent calculation om forests. We now see that the data from forests is clustered and almost totally 'ed from the data for cities, represented by the triangles. Even though a : ints still overlap, the improvement is very dramatic. The transformation has succeeded in reducing the dimensionality of the original feature vector from 13 to 2 and also in separating the clusters of data for the two classes.

Figure 3 shows the results of the transformation for the two classes of cities and fields. The x's represent the data from cities. Good separation is obtained for the two classes as only a few points are overlapping. Figure 4 shows the results of calculations for the two classes of cities and water. The x's represent the data from cities. In this particular case the two classes are totally separated as well as being clustered fairly well. Figure 5 presents the results for forests and water. The x's represent the data for forests. Again the two classes have clearly been separated by the transformation. Figure 6 shows the results for fields and water, with the data for fields represented by the x's. These two classes have also been totally separated. Figure 7 shows the results for forests and fields with the data for forests represented by the x's. The two classes are not well separated and other features may have to be investigated to obtain better separation. Another possibility would be to try using nonlinear feature selection techniques for these two classes.

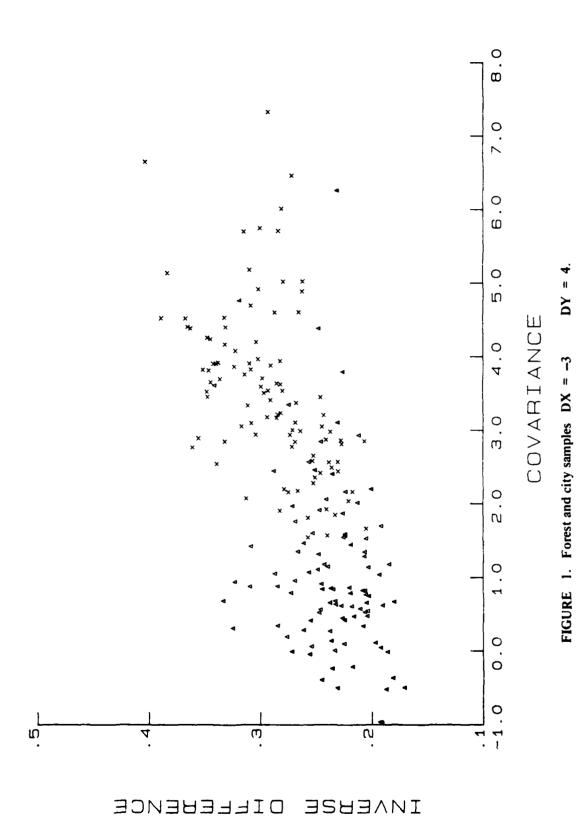
²This assumption of equal probability for cities and forests appeared to be appropriate for the area where the samples were taken.

TABLE 1. Values of J_1 for first and second order histogram statistics

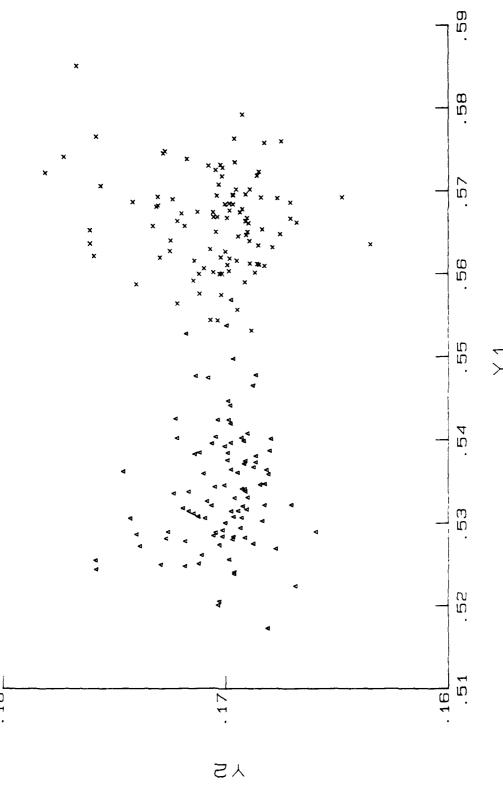
DX	DY	J
1	0	22.1485
2	0	19.8078
3	0	19.4333
4	0	19.3099
5	0	19.7508
6	0	20.3677
7	0	19.8529
0	1	21.0314
0	2	21.7613
0	3	21.3524
0	4	21.7343
0	5	22.3144
0	6	23.1462
0	7	23.5903
1	1	20.6771
1	2	20.4869
2	2	24.3979
3	2	23.9334
4	2	21.5577
3	3	25.7921
1	1	20.2745
2	2	24.4373
3	2	25.0272
3	4	29.8525
4	4	22.7131

CONCLUSIONS

- 1. The feature selection technique discussed in this report appears to be a powerful tool for the application of pattern recognition to high resolution synthetic aperture radar imagery.
- 2. In order to separate forests and agricultural fields it appears that the first and second order histogram statistics combined with a linear feature selection technique may not be sufficient as a feature vector.

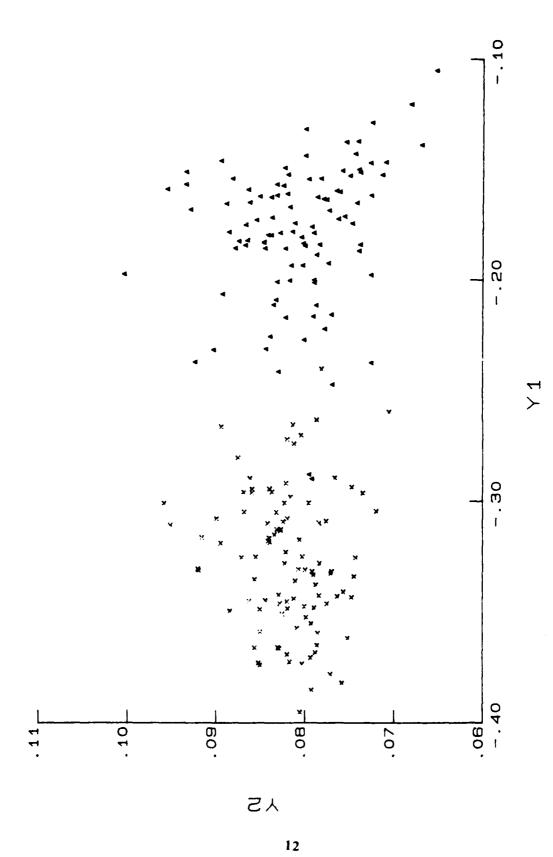






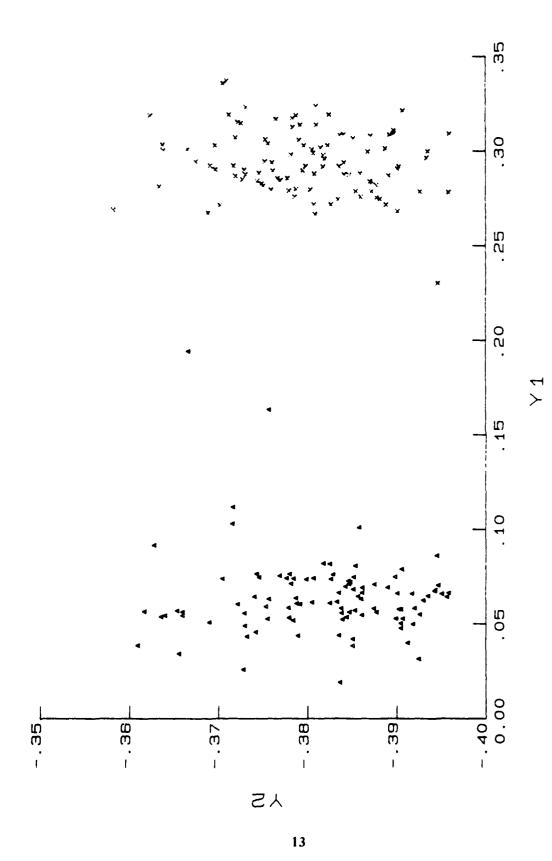
DY = 4.

FIGURE 2. Forests and cities DX = -3



DY = 4.

FIGURE 3. Cities and fields DX = 2



DY = 0.

FIGURE 4. Cities and water DX = 1

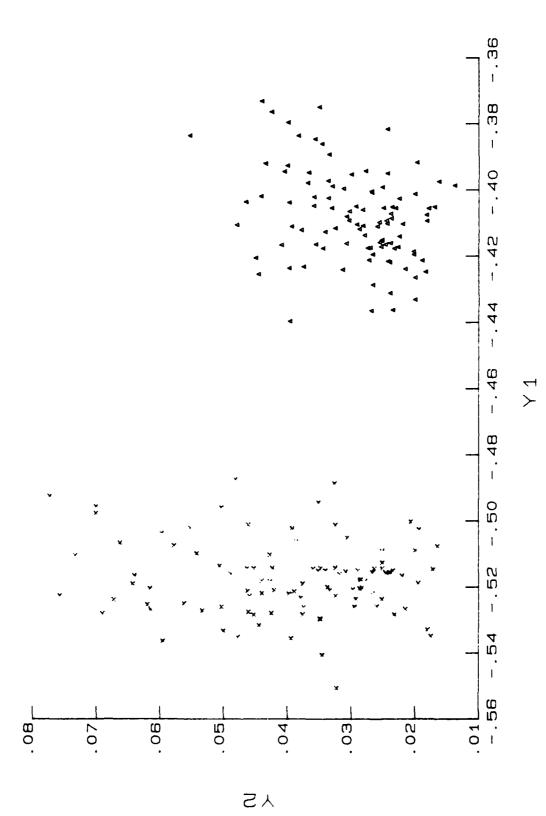
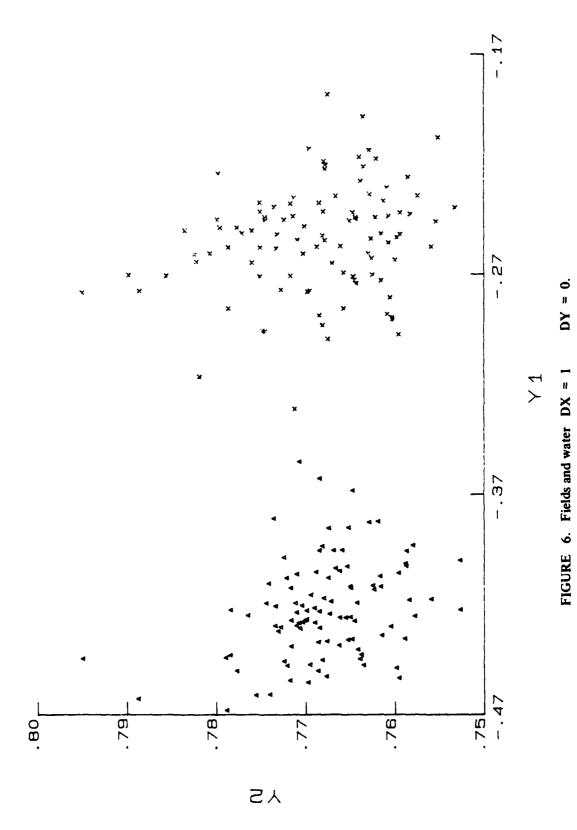


FIGURE 5. Forest and water DX = 1 DY = 0.



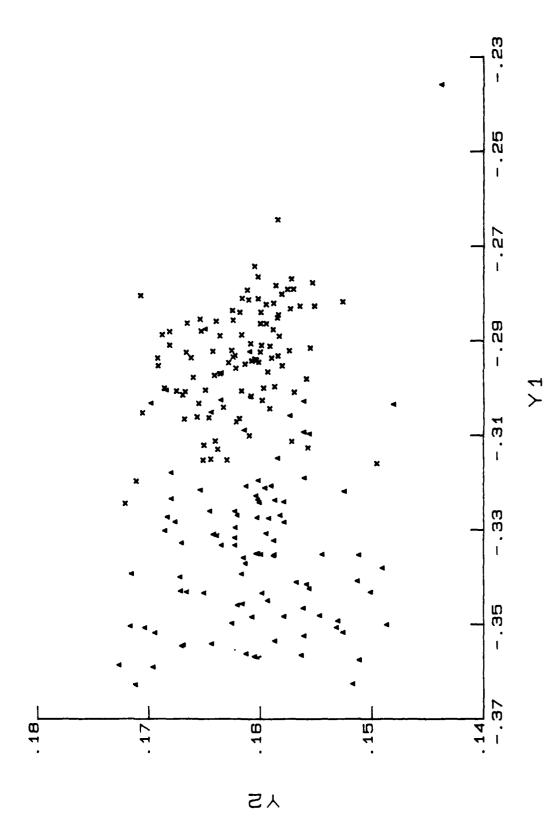


FIGURE 7. Forests and fields DX = 2 DY =

APPENDIX A

FEATURE VECTOR COMPONENTS

The following first and second order histogram measures were used to construct a thirteen dimensional feature vector:

Variance
$$\sigma_b^2 = \sum_{b=0}^{L-1} (b-\overline{b})^2 P(b) = x_2$$

Skewness
$$b_S = \frac{1}{\sigma_b^3} \sum_{b=0}^{L-1} (b-b)^3 P(b) = x_3$$

Kurtosis
$$b_K = \frac{1}{\sigma_b^4} \sum_{b=0}^{L-1} (b-b)^4 P(b) - 3 = x_4$$

Energy
$$b_N = \sum_{b=0}^{L-1} \{P(b)\}^2 = x_5$$

Entropy
$$b_E = -\sum_{b=0}^{L-1} P(b)\log_2[P(b)] = x_6$$

Autocorrelation
$$B_A = \sum_{a=0}^{L-1} \sum_{b=0}^{L-1} abP(a,b) = x_7$$

Covariance
$$B_C = \sum_{a=0}^{L-1} \sum_{b=0}^{L-1} (a-a)(b-b) P(a,b) = x_8$$

Inertia
$$B_{1} = \sum_{a=0}^{L-1} \sum_{b=0}^{L-1} (a-b)^{2} P(a,b) = x_{9}$$

Absolute Value
$$B_V = \sum_{a=0}^{L-1} \sum_{a=0}^{L-1} |a-b| P(a,b) = x_{10}$$

Inverse Difference
$$B_D = \sum_{a=0}^{L-1} \sum_{b=0}^{L-1} \frac{P(a,b)}{1 + (a-b)^2} = x_{11}$$

Energy
$$B_N = \sum_{a=0}^{L-1} \sum_{b=0}^{L-1} [P(a,b)]^2 = x_{12}$$

Entropy
$$B_E = \sum_{a=0}^{L-1} \sum_{b=0}^{L-1} P(a,b)\log_2 [P(a,b)] = x_{13}$$

where L is the number of grey levels and P(b) and P(a,b) are given below

$$P(b) = \frac{Q(b)}{M}$$

M is the total number of pixels in the sample window. In this case M was equal to 1024. Q(b) is the number of pixels of greytone b which occur in the sample window.

$$P(a,b) = \frac{Q(a,b)}{M}$$

Q(a,b) is the number of times greytone a is located next to greytone b by the displacement Δx and Δy .

APPENDIX B.

Computer Program for Calculating J, and the Transformation Matrix A

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              WRITE(6.2001)3
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             WRITE (6.140 -
0.084
         148 FORMATOZX "COVARTANCE MATRIX")
0085
             DU 142 K 1.15
0.086
111181
              WRITE (6 144) (L(R M.NJ) M-1 13)
         144 FORMAT (1X.73(F8.4.18))
0088
0.069
         142 FUNTINUE
0090
          80 CONTINUE
0051
         150 DU 152 NJ-1 NC
0.092
              WR) 1E (EU. 154) NJ
         154 FURMATC"NJ:" 11.2% "ENTER THE APRIORI PROBABILITY OF THIS CLASS")
0093
01194
              READCLU.280P(NT)
0095
          28 FURMAL(F10.8)
             WRITE (6.156)NT, P(NT)
0096
         156 FURMATCIA "NJ=" 11 5x "P(NJ) =" 110.8)
0000
         152 CONTINUE
0098
0099
             OH 90 J=1.NC 1
0100
             DO 90 Fal+1.NO
010)
          로# 5군(] [])=#,#
             DO 72 1-1 NC-1
0102
              Dir 72 Jal+1 NC
0103
              DO 72 M-1.15
1111114
              32(1 J)=52(1 J)+(A(M I) A(M J))##2
01.05
          25 CONTINUE
0106
0107
              DR 95 1=5 NL 1
0.008
              DO 91 J-1+1.NC
0149
               æ(1.J)≈SWR1(S2(1.J))
          91 CONTINUE
0110
01.1
             WRITE(6.73)
          73 FORMATCIX. "THE INTERSET DISTANCES")
011,
             1-14, (=1 SP 0d
24 (+1-1 SP 0d
0115
0114
          WRITE(6.74)1.3.52() 3)
74 FORMAT(3X "I=" 3) 5X."J=".[1.5X "D(1.3)=" L)5.8)
0435
0116
0117
          AS CUNTINUE
0116
              po 30 1=1.13
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0119
             DO 30 J=1.13
0.120
             SZ(1,J)=0.0
0121
          30 St(1.J)=0.0
0122
             00 32 741 .13
0123
             DO 32 J=1 13
0324
             DO 32 K=1.NC
          32 S2(1,J)=$2(1,J)+$(1,,K)*P(K)
0.125
0126
             DO 34 J=1.13
01.27
          34 XmO(J)=0.0
0.158
             DO 48 J=1.13
0129
             DO 40 M=1.13
0130
          4U S1(J.N)=C(J.M.1)+C(J.M.2)+(A(J.1)-A(J.2))*(A(M.1)-A(M.2))
03.31
             DU 160 J=1.13
0132
             00 160 3-1 (13
0)35
             K: 1+13*(J:1)
0134
             R(K)=St(I,J)
01.35
             B(K)=$2(I,J)
0136
         160 CONTINUE
03.32
             DU 41 1=1.13
0.138
          41 XMU(1)=0.0
01.59
             CALL NROUT(13.R.B.XMU.ET)
9146
             DU 165 J=1.13
0141
             00 165 1=1 .13
             K-I+13*(3 1)
0142
0143
             S2(L.J)#ET(R)
0144
         165 CONTINUE
0145
             WRITE (6.42)
11146
          42 FORMAT(2X: "ETGENVALUES")
0147
             WRITE(6.46)(XMO(1).5=1.13)
0148
          46 FURMAT(1X.13(E9.3.1X))
0149
             WRITE(6.48)
          48 FORMAT(2X. "EIGENVECTORS")
0150
0151
             DO 50 T=3.13
             WRITE(6.76)($2(1.J).J≈1.13)
0152
0153
          76 FURMAT(1X.13(F8.5.1X))
0154
          SU CONTINUE
0155
          64 WRITE(LU.52)
8156
          52 FORMATC"ENTER A VALUE FOR MICELINC")
0.152
             READCLU. 49M
0158
             X\cup \mathbf{1}=U\cup U
0159
             DO 54 L-1.M
0160
          54 XJ1#XJ1+XMQ(I)
02.61
             WRITE(6.56)XJ1.M
          SK FORMAT(2X."THE VALUE OF Jim".Fig.4.5X."Mm".31)
0162
             CALL IRMAT($2,51.13.13.0)
00.63
0164
             WK LIE (6.58)
          58 FORMAT(2X. "THE TRANSFORMATION MATRIX A")
11165
             DO 60 1=1.M
0166
0167
             WRITE(6.46)(S1(1.3),J=1.13)
0168
          60 CONTINUE
0169
             WRITE (LU.62)
0170
         62 FORMATC"DO YOU WANT TO CHANGE THE VALUE OF M? IF YES TYPE 1 IF
0171
            INO TYPE 2")
0172
             READ(LU.4)17
0173
           4 FURMAT(11)
             1F (12.EQ.1)GO TO 64
0174
0175
             WRITE (UU.82)
0176
         82 FORMATC "DO YOU WANT TO CHANGE THE VALUES OF THE APRIORI PROBABIL
0177
            FITTES? IF YES TYPE LIFE NO TYPE 2")
             READ (LU. 4) TAZ
0178
```

```
0179
                TECJAZ.EQ. 1360 10 150
0180
               WRITE(LU.68)
8181
            AS FURMATO DO YOU WANT TO CHANGE THE VALUES OF TOX AND TOY? IF YES
              STYPE 1, IF NO TYPE 2.")
03.82
0183
               READGLU, 4) 1XZ
               11 (1XZ.EQ. 1)CO 10 1
0184
          499 510F
0.185
               FND
0186
0.187
               SUBROUTINE COVERCKK COV AVININI * PS)
មានម
               DIMENSION X(100-13) ((13.13) (A(13) (2(100.13) CUV(13.13.2), AV(13
0.189
              1.2).VAR(13)
0190
               KK≃KK
9391
               DO 50 K-1 N
               ACK ) = U . U
11142
               DU 45 J=1.88
0193
01.94
           45 A(K)=A(K)+X(J,K)
11194
           50 A(K)=A(K)/AL
0196
               DU 55 K-1.N
0197
               VAR (K)=0. II
0198
               DU 54 1-1 KK
1, 7 34
           SA VARCIOSOVAR (ROFICX () KO MIKO) **
0.020
           SS CONTINUE
0.200
               190 AU K - 1 W
           60 VAR(K) VAR(K)Z(KK-) -
0.20.2
0.20.5
               0.04 \pm 0.04
11261
               DH 45 K- 1 N
0.205
           65 03 D3+VAKCKI
02116
              03-2. *D3
0207
               DI 110 K-1 N
0.208
               DU 115 151.66
\alpha_{2} \otimes \beta
          111 / (1 R) - X(1 R) - A(R)
0 - 10
          THE LIGHT FAIL
W^{N} \leftarrow Y
               DE 129 Kor N
11,217
               DIL 125 M. 1 14
41,23 6 5
               LUCK (N.) II., G
              DU 130 1 - + ks
0.214
          A SHELL CLE ME LOCK MIRE CLE KERS CLE ME
16 15 15
11.216
              LICK MINER BOY (K
          TO CHALLNIE
4. 4.
11.-184
          120 LUNIINUE
0. 🕶
              100 135 K + N
0.2,26
               DIL 135 M. I N
              WILL NIDEACK
67.71
          135 THUCK M NTO THE ME
0,2,2,7
              ar Irach
M_{n-1} \cap A
11.2.23
              F ND
               STOREMENTAL MEMBERS OF A STATE OF
0.5775
               BOTH MESTING ACTIONS FOR CLASS FOR CONST.
D . 69
0.5.75
0.1 1.
               por right the re-
               i, nacl (i)
£1 , 1.₩
               001 100 1-1 2
H_{c} = xH
07:35
              1 1 + 1
0.57
              K 1, + 1
         100 ROKE-RULE
1177 × 5
117 × 54
6,750
              TALL FRANCES OF MUS
dr. Str.
              4 - 44
H_{\mathcal{F}} \to \mathbb{C}^*
              441 1111 1 2 2
50 To Ke
              2 9 9 9
```

```
TIBLALIDET, BYSURTCARS(RC()))
0.239
0240
                                                                   k ≒ti
                                                                    DO 115 Jea M
024)
                                                                   DU 115 L:1.M
0242
                                                                   K - K + 1
0243
                                              115 R(K)=X(K)*X((J)
0244
0245
                                                                   भाग १८४ । । भ
0246
                                                                   N^{2} \sim 11
                                                                   .
β(1 1,2∦ 1 π <sub>29</sub>
0247
0248
                                                                   N1--M*(1 1)
0249
                                                                   1 - 14 (3 1) (3
0250
                                                                    A \in E \to \{0\}, U
                                                                   title 1. 'U. K. . . M
0.351
0.05.0
                                                                   NI di+i
0.255
                                                                   18,7 - 18,7+3
                                              120 2017-2017-4017-1817-1817
0244
41,055
                                                                  5 - FF
0258
                                                                   QU 130 F 1 0
41, 24, 27
                                                                   District State of the
0,253
                                                                   141 1-19
4.24.4
                                                                   N. MELLINE
0.260
                                                                  1 1 + 1
H_{\rm s}^{-2} to A
                                                                   Section 0 to
                                                                   DH 138 K 1 M
11.26
                                                                   121 - N1 + M
11,365
                                             No objets
1 All Green Garrier Charles Charles
1 All Egitte Wille X (N. 2005)
0264
0.155
0.266
                                                                   1 11
Bons
                                                                   July 140 1 1 10
11268
0.269
                                                                  1 ( + )
                                              144 WILL ALL ...
0,270
                                                                   0.224
                                                                   plata in
H_{\star} = \ell_{\star}
                                                                    \mathbf{e}((1-\mathbf{j})^{T},(0-r-r-\tilde{r}))
\{u_{i,j}\}_{j=1}^{n}
W_{1} \leq A
                                                                   14.1 1 - 71
 D. . . .
                                                                   A CHARLEST AND A
11,27776
                                                                   \alpha(1) + \alpha(0)
11 1
                                                                    out that I say
 a_{i}/C_{i}
                                                                    141 1111 + 15
 M. 759
                                                                     11 W. 3 + 3
 0.000
                                                Part service and expenses the transfer
  Beech
                                                              ‡ - H
11 2.2
                                                                   8 H
  H_{\alpha}(r,\beta) \propto
                                                                     pot radii a se se
 4. 114
                                                                      a(eV_2 - H - H)
 H,2863
                                                                     policy that the con-
11. 7 /,
                                               \chi(\mathcal{M}_{B}) = \operatorname{Section}(\mathcal{M}_{B}) = \chi(\operatorname{Best *} \star \operatorname{const.}) + \operatorname{december *} \star \operatorname{december *} \times
  4.27
                                                                      manufacture of the contract of
al sit.
 0.0352
                                                                    note profit I be of
  a - 5 H
                                                                    e 1. i.,
  0-25
                                               matter of Kill Perkins of the
                                                                     . I may
 0.000
                                                                   1.41
11 ...
                                                                            was a constructed to be a constructed to the contract of the contract of
                                                                rish a caper see raw ones a checaphic
n \sim 2
                                                                       Compact professional Communities (4)
                                                                                                                                                                                                                                                          Trickley a chair eye is a
                                                                        Contract to the contract of th
```

```
0299
                                        VAR = 0 . 0
 0300
                                         SKW≃0.0
 0301
                                        XK 1=0.0
 0302
                                        FNG=0.0
 0303
                                        EN1=0.0
 0304
                                        AU1=0.0
                                        COV=0.0
 0305
 0306
                                        X1E=0.0
                                        ABS5=0.0
 0.307
 0.508
                                        XID=0.0
 0309
                                        ENY=0.0
 0310
                                        ENR = 0.0
 0.411
                                        DO 10 J=1.32
 03%
                                        56.14 01 00
 0.51.5
                                        M -J -1
                              10 TAR(1.J)=[MAGE(1+M#32)
 0314
 0315
                                        M 1024
                                        DO 20 K#1 NGKAY
 0316
 0.317
                               20 CONT(K)=0.0
                                        00 30 1=1.32
00 30 J-1.32
 03.08
 0319
                                         IR-FARCE JO
 63.46
                                        CONFICER+I) CONFICER+1)+1.
 0374
                               30 LANTINEL
 0.370
 0.325
                                        Xm∍m
 11324
                                        00 32 K-1 NGRAY
 0.525
                                        DO 32 MAI NIGHAY
 0326
                               SE COULLOK MYSO . U
                                        MX: 32+10X
 0300
                                        TE (32-MX)12 (14.16
 0323
                               12 INCH-MX-32
 ひふごと
 0.330
                                        RXBel
 0331
                                        NXF -3e-ENCH
 0330
                                        (11 10 19
                               14 NXH-1
 0.5,5.5
 03 44
                                        WXL - Se
 11.4 45
                                       GO TO 42
 0.536
                               16 (N) R-12-MX
 0.337
                                        NXH-1+IN/ h
 0336
                                        11XN - 55 - 1XN
 1),4.49
                               19 147 32+101
 0.540
                                        11 (32 M) 101 (3125
                              21 INCREMY-32
 0341
 4 5 4
                                        (S^{k})_{k}(Y) = Y
                                        MIE 43 INCh
 0345
11:44
                                       111 111 25
 0.345
                               23 NYRSI
 0.346
                                       1851 - 30
 11347
                                       60 TO 21
                              POST OF STREET
 11.5.42.
 11 445
                                        MINI+1 HIN
 14-53-54
                                        William Assembly
 0.35sa
                               22 CONTINUE
                                       041 35 1 4 MAG
11.35
11 5 n n
11 5 l maj
                                         CHERRY COLUMNICS OF
Hast.
                                        LO JARCHIERE TRIDE WERE CITATION
Harrists.
                                       CONTRACTOR OF A CONTRACTOR OF THE STATE OF T
0.5%
                               A CHARLING
0.85.03
                                       त्रात का एक अधिका
```

```
40 P(K)=CONT(K)/XM
6.359
             DO 45 K=1 NGRAY
0.360
          45 AUG: AVG+(K-1)*P(K)
0.361
             DO SO KELLNGRAY
0.362
          50 VAR=VAR+((K-1-AVG)**2)*P(K)
0364
0364
             SIDESORT (VAR)
             DO 55 K=1 NGRAY
0.365
          55 5KW#5KW+((K-1-AVG)**3)*P(K)
0366
             SKW=SKW/STO**3
0.367
             DU 60 K-1 NGRAY
0.366
          60 XKT=XKT+((K-1-AUG)**4)*P(K)
0369
0320
             XK1=(XK1/(SID##4))-3.
0.571
             DO 65 K-1 NGRAY
0.372
          65 ENG=ENG+P(K)#P(K)
0.373
             DO 70 K=1.NGRAY
0374
             1E (P(K).EU.0.0)GO TO 70
0.575
             ENT=ENT+3.321929#P(K)#ALOGT(P(K))
0376
          ZU CONTINUE
0,327
             ENT =-ENT
0378
             DO 75 K=1 .NGRAY
0379
             DO 75 Mad INGRAY
0.380
          75 PP(K.M)=COULL(K.M)/XM
             DU 80 K-1 NGRAY
0381
             DO BU MES NGRAY
0.382
             AUT =AUT+(K-1)*(M-1)*PP(K.M)
0.383
0.384
          80 CONTINUE
0.385
             AUGK # 0.0
             AUGM=0.0
0.386
0.387
             DO 85 KHILNGRAY
             DO 85 M-1 NGRAY
0388
             AUCK=AUCK+(K-1)*PP(K.M)
0.389
          85 AUGM=AUGM+(M-1)*PP(K.M)
0.399
             DO 90 K-1 NGHAY
DO 90 M-1 NGRAY
0.391
0392
0393
          90 XJE=XIE+(K-M)*(K-M)*PP(K,M)
0.394
             DU 95 K-1.NGRAY
             DU 95 Mas NGRAY
0395
0396
             ABSS=ABSS+(JABS(K-M))*PP(K.M)
0.397
          95 CONTINUE
0398
             DO 100 K=1.NGRAY
0399
             DO 100 M=1.NURAY
0400
         108 XID=XID+PP(K.M)/(1.0+(K-M)**2)
0401
             DU 105 Kat .NGRAY
0402
             DO 105 M=1 .NGRAY
0403
         105 ENY HENY+PP(K,M)*PP(K,M)
0404
             DU 110 K=1 NGRAY
             DO 110 M=1.NCRAY
0405
             IF (PP(K.M), EQ. 0.0)GO TO 110
0406
0407
             ENR=ENR+3.321929*PP(K.M)*ALOGT(PP(K.M))
0408
         110 CONTINUE
             ENR =- ENR
0409
             DO 115 K=1 NGRAY
DO 115 M=1 NGRAY
0410
0411
0412
             COV=COV+(K-1-AVGK)*(M-1-AVGM)*PP(K,M)
0413
         115 CONTINUE
0414
             RETURN
             END
0415
0416
             LND$
0417
```

APPENDIX C.

Computer Program for Calculating the Transformed Feature Vector Y = AX

```
ATAXIR THUUUU4 IS ON CRUUU11 USING 00013 HCKS R#0000
      0002
0003
        THIS PROGRAM COMPUTES THE TRANSFORMATION
        Y=AX WHERE A IS COMPUTED FROM ANOTHER
0004
        PROGRAM AND IS INPUT HERE
0005
     0006
0007
           PROGRAM YAXIR(3,1000)
           DIMENSION LUOT(5).IMAGE(1024).X(100.13).Y(100.2).A(2.13)
0008
0009
           DIMENSION IDCB(144) IFILE(3) AM(2) VAR(2)
           DIMENSION AV(6.6),D(2.2)
0010
0011
           EQUIVALENCE (IMAGE.Y)
0032
           CALL RMPAR(LUUT)
0013
           FU#EUUT(1)
0014
           CALL ERLU(LU)
0015
           NC#2
0016
         4 FORMAT(I1)
0017
           WRITE(LU,6)
         6 FORMATC "ENTER THE VALUE FOR MITHE NUMBER OF ROWS IN THE TRANSFOR
0018
           IMATION MATRIX A")
0019
0020
           READCLU. 4) M
0021
           WRITE(LU,8)
0022
         B FORMAT("ENTER THE TRANSFORMATION MATRIX A")
0023
           DO 10 1=1.M
           DU 10 J=1.13
0024
0025
           WRITE(LU,12)I.J
        12 FORMAT("]=".12.2X,"J=".12,"A(T.J)=")
0026
0027
           READ(LU,14)A(1,J)
0028
        14 FURMAT(E9.3)
        10 CONTINUE
0029
0030
           WRITE(6.29)
0035
        29 FORMATCIX. "THE TRANSFORMATION MATRIX A")
0032
           DO 78 I=1.M
0033
           WRITE(6.77)(A(1,1),J=1,13)
0034
         77 FORMAT(1X.13(E9.3.1X))
0035
         28 CONTINUE
0036
           DO 80 NJ=1.NC
0037
           WRITE (LU.16)
0038
        16 FORMAT("ENTER THE NUMBER OF IMAGE SAMPLES TO BE ANALYZED .LE.100")
0039
           READ(LU.18)NUATA
0040
        18 FORMAT(I3)
0041
           WRITE(LU, 20)
        20 FORMATC "ENTER THE FILE NAME FOR THE DATA SET")
0042
0043
           READ(LU\22)IFILE
0044
        22 FORMAT(3A2)
0045
           WRITE(LU.15)
0046
         15 FORMAT("DISK LU NUMBER?")
0047
           READOLUS 21010 LDLU
0048
           WRITE(LU.2100)
0049
      2108 FORMAT("ENTER A VALUE FOR IDX")
0050
           READ(LU.2101)TDX
0051
      2101 FORMAT(12)
0.052
           WRITE(LU.2200)
0053
      2200 FORMAT ("ENTER A VALUE FOR TDY")
0054
           READ(LU:2101)IDY
0055
           WRITE(6.1400)1DX,1DY
      1400 FORMAT(2X,"[DX=",12,5X,"TDY=",12)
0056
0057
           WRITE(6,200)(FILE
0058
       200 FORMAT(1X,3A2)
```

```
0059
             CALL OPENCIDOB JERR JETLE . 0 , 0 .- IDLU)
0060
              IFCTERR.LT.00GO TO 2000
0061
             60 TO 24
0062
        2000 WRITE(LU.2010) TERR
        2010 FORMATC "OPEN FILE ERROR" (15)
0.063
             GO TO 999
0064
          24 1 CONT=1
0065
             CALL LABIN(TDC8.6)
0066
          13 J=1
0067
0068
             DO 19 T-1.8
             CALL READF (IDCH. TERR (IMAGE(J))
0069
          19 J=J+128
0070
0021
             JECJERRALI, 00GO TO 3000
             6S 0T 00
0.025
        3000 ICONT=NDATA
0073
             WRITE(LU.2020) TERR
0074
0075
        2020 FORMAT("READ FILE ERROR" (15)
0076
             GO TO 999
0077
          26 NGRAY=16
             CALL ISCAL CIMAGE (IMAGE (1024 (0 (15)
0078
             CALL FEVEC () MAGE, AVG, VAR, SKW, XKT, ENG, ENT, AUT, COV, XIE, AHSS, XID,
0079
0080
            LENY, ENR. NGRAY, LDX. LDY)
0081
             X ( ) CONT ( ) ) = AVG
0082
             X(ICONT.2) = VAR
0083
             XCLCONT(3)=5KW
             X ( LCUN1 . 4) -XKF
0084
0085
             XCLCONT.5) = ENG
             XCICONE.6) = ENF
0086
0.087
             X (100NT JY) ≃AUT
0088
             XCICONT.80#C09
0.089
             XCLCON1.9)~XIE
0090
             XCICONI.10) =ARSS
             ACICONI (11) - XID
0.091
0092
             X(ICUNT, 12) #ENY
             X(1CON1.13)=ENR
0093
BH94
             TECTCONT-NDATA)28,40.30
0095
          28 (CONTELLONIE)
0096
             60 TU 13
0.097
          SHEWRITE CLUSSONS STETLE
          32 FORMAT("NJ#" ()1,3X.3A2)
0098
0099
             DH 34 J-1.NDATA
03.00
             DO 34 K*1.h
0101
          34 Y(1,K)=0.0
             DO 36 F-1 NDATA
0102
0105
             DO 36 K-1 M
0104
             DO 36 MK=1.13
0105
          36 YOF KINY (1.K)+A(K,MK)*X(),MK)
             CALL GPLOICYCLID, YCL, 20, 100 LH, NJO
0106
             90 SU K-3 M
0107
0108
             AM(K)=U, fi
0107
             KK-NDATA
0110
             DO 45 J. L. NDATA
          45 AM(K)-AM(K)+Y(J,K)
0111
          SU AMIKO -AMIKO ZXK
0332
             DO 55 K - 1 . M
0115
0114
             VAR (K) -U, II
0115
             00 54 1-3 NDATA
          54 UAR(K): VAR(K)+(Y(1,K)-AH(K))**./
0)16
          SS LUNTINIII
0117
0118
             DO 68 K-1.M
```

Section Sections

```
0119
               60 VAR(K)=VAR(K)/(XK-1.)
                     D3=0.0
                PU 65 K=1.M
65 D3=D3+VAR(K)
0121
8122
               65 D.3=D.3+VAR(K)
D.3=2, #D.3
D.0 70 K=1, M

70 AV(K,NJ)=AM(K)
80 CONTINUE
CALL GPLOT(Y(1,1),Y(1,2),~100,~LU,2)
D.0 81 X=1,NC-1
D.0 81 J=J+1,NC
81 D(1,J)=0.0
D.0 82 T=4 NC=4
0123
8125
0126
0127
9520
0129
0130
                     0131
0132
0133
                     DO 82 K=1,M
               82 b(1,3)-4(1,3)+(A)-(A)-(A)-(A)-(A)-(B) 083 I=1.NC-1
0134
0135
0136
                     DO 83 J#1+1 NC
                83 D(i,J)=SGRT(D(L,J))
8137
               83 D(1,3) #SBR((D(1,3))
WRITE(6,87)
BY FORMAT(1X, "THE INTERSET DISTANCES")
DO 86 I=1,NC-1
DO 86 J=1+1,NC
WRITE(6,84)I,J,D(1,J)
84 FORMAT(1X, "I=",I1,2X, "J=",I1,2X, "D(1,J)=",E15,8)
0138
0139
0140
0141
0143
0144
               86 CONTINUE
0145
              CALL CLOSE (IDCH)
999 STOP
0147
                     END
0148
                     ENDS
```

